FINITE ELEMENT ANALYSIS OF OPTICAL WAVEGUIDES

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Summary

A finite element program for the analysis of anisotropic optical waveguides is described. The appearance of spurious numerical modes due to the fact that the functional is non positive definite is discussed and a possible solution to the problem is presented.

Introduction

In the field of optical communications monomode or quasi monomode guides have become important due to the growing interest in single mode fiber and integrated optical waveguide structures. The analysis of such waveguides is not an easy problem since in general the geometry can be quite complicated and the materials anisotropic. The finite element method is probably the waveguide analysis method that is the most generally applicable and most versatile. Once a finite element program has been written any geometry and material combination that can be suitably represented by a division in triangles can be analysed. Although the finite element method has been used for the eigenmode analysis of dielectric waveguides for more than 10 years[1], its application remains rather difficult. In this paper, the main problems associated with the finite element approach are discussed and solutions for some of those problems are proposed. In the next paragraph we consider the most general case : an anisotropic guiding region of arbitrary cross-section, index variation and an anisotropic substrate region. In this case a full vectorial analysis is necessary and a suitable variational principle is presented. The appearance of higher order spurious numerical modes is discussed and a possible solution to that problem is proposed. In the case of isotropic waveguides two approximate scalar finite element analysis methods are presented. The accuracy of this method is discussed and the very important computational advantages of this approach are illustrated by a number of examples.

Anisotropic waveguides

In integrated optical devices that contain electro optic or elasto optic interactions, optical waveguides are made on crystal substrates. This means that the eigenmode analysis method has to be able to handle anisotropic guides. If the crystal has a diagonal permittivity tensor one can rewrite Maxwell equations in terms of the longitudinal components E_z and H_z in the following way :

$$-\left[\frac{\partial}{\partial x}\left(A_x\varepsilon_x \frac{\partial E_z}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_y\varepsilon_y \frac{\partial E_z}{\partial y}\right)\right] + \frac{\beta}{\omega}\left[\frac{\partial}{\partial y}\left(A_y \frac{\partial H_z}{\partial x}\right)-\frac{\partial}{\partial x}\left(A_x \frac{\partial H_z}{\partial y}\right)\right] = \varepsilon_z E_z$$

$$\mu_o\left[\frac{\partial}{\partial x}\left(A_y \frac{\partial H_z}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_x \frac{\partial H_z}{\partial y}\right)\right] + \frac{\beta}{\omega}\left[\frac{\partial}{\partial y}\left(A_x \frac{\partial E_z}{\partial x}\right)-\frac{\partial}{\partial x}\left(A_y \frac{\partial E_z}{\partial y}\right)\right] = -\mu_o H_z$$

where $A_x = \dfrac{1}{k_o^2 \frac{\varepsilon_x}{\varepsilon_o} - \beta^2}$, $A_y = \dfrac{1}{k_o^2 \frac{\varepsilon_y}{\varepsilon_o} - \beta^2}$

k_o = wavenumber in vacuum

β = wavenumber of the guided mode.

The finite element formulation is based on following variational expression for the previous equations :

$$\delta L = 0$$

$$L = \iint \frac{1}{2}\left[-\varepsilon_z E_z^2 - \mu_o H_z^2 + \varepsilon_x A_x\left(\frac{\partial E_z}{\partial x}\right)^2 + \varepsilon_y A_y\left(\frac{\partial E_z}{\partial y}\right)^2 + \mu_o A_y\left(\frac{\partial H_z}{\partial x}\right)^2 + \mu_o A_x\left(\frac{\partial H_z}{\partial y}\right)^2\right] + \frac{\beta}{\omega}\left[A_x \frac{\partial E_z}{\partial x}\cdot\frac{\partial H_z}{\partial y} - A_y\frac{\partial E_z}{\partial y}\cdot\frac{\partial H_z}{\partial x}\right] dS \quad (1)$$

If the waveguide is isotropic, equation (1) is reduced to the well known functional discussed for example in references [1], [2] and [3]. Starting from equation (1) a general finite element program for the analysis of anisotropic optical waveguides has been written. When using such a program one is faced with a number of problems and trade-offs.

1. The choice of the type of elements and the number of elements needed to model the waveguide. The most simple triangular element assumes a linear interpolation between the field values at the corner points of the triangle. Using this type of element one obtains large but sparse matrix equations. By carefull numbering of the nodal points, band matrices can be obtained. Instead of the linear elements, one can also use triangular elements with higher order polynomial interpolation functions. The drawback is that the programming effort for those higher order elements is quite large. The advantage is that, one can obtain accurate results with much smaller matrix dimensions. We have found for

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example that in the case of a rectangular overlay guide, where a small number of triangles is sufficient to model the geometry, one obtains about the same accuracy with 900 linear elements or 928 nodal points as with only 9 fourth order elements or 87 nodal points. Since the geometry or index variation of some guides can be so complicated, as to require a large number of triangles, the finite element program allows one to specify the desired element order between 1 and 4.

2. The modelling of the infinite transverse extent of the waveguide always represents a problem. Three possible solutions are : imposing an artificial zero boundary condition for E_z and H_z at a large enough distance from the guide, use sector elements[3] that assume some exponential decay for the field or implementing the radiation condition through an integral equation at the boundary of the finite element region. The last method, although exact, leads to such a complicated set of equations, that it is numerically impractical to use. The sector elements would be ideal if one could find the exponential decay factor, as a result of the variational process. Since this leads to non linear equations one has to determine the best exponential decay by trial and error for each point on the dispersion characteristic. The first method has as advantage its simplicity. It has been used for the calculation of the results presented in this paper but care has been taken to make sure that the influence of the position of this zero boundary condition on the obtained results, was negligible.

3. The most serious diificulty in using the finite element analysis, for open dielectric waveguides, is the appearance of spurious, non physical modes. This means that a number of the eigenvalues and eigenvectors of the matrix eigenvalue problem, do not represent physical modes of the waveguide, but are spurious results introduced by the numerical technique. The reason for the appearance of the spurious modes is the fact that the functional (1) is not positive definite since A_x or A_y can be positive or negative, depending whether the element is in the guide or in the substrate. If one is interested only in the calculation of the lowest propagating mode, the appearance of those spurious modes is not much of a problem. However, if one wants to compute a set of higher order modes, it becomes very difficult to distinguish between the spurious and the physical modes of the guide. However, we have found that by explicitely enforcing the continuity of the tangential components of the transversal fields at the interfaces, by means of Lagrange multipliers, most of the spurious modes disappear. This can for example be seen on figure 1. The circles represent all the solutions of the classic finite element program, while the results of the finite element program with continuity conditions are indicated by crosses. One can clearly see that all the results of this last program, lie on the dispersion characteristic of the modes of the guide. The disadvantage of this solution lies in the greatly increased complexity of the program and of the numerical operations that have to be done for enforcing those continuity

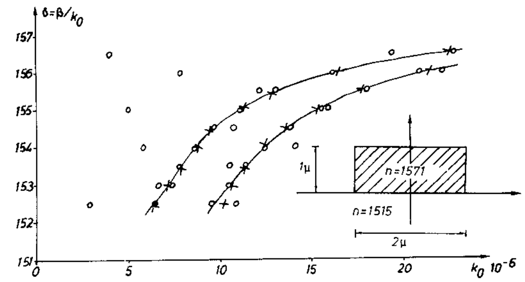

Fig. 1 Dispersion characteristic of a rectangular overlay guide

conditions. For very complicated guide geometries for example the accumulation of rounding errors becomes a problem. If the guide is isotropic or if it can be approximated by an equivalent isotropic guide, we propose in the next paragraph an approximate finite element formulation that allows a much easier and faster calculation of the different modes of the guide.

Approximate scalar finite element formulations

If the optical guide is isotropic, we propose two different scalar formulations, that yield excellent approximations for the EH and HE type of mode of an integrated optical waveguide. As an example we consider a rectangular overlay waveguide with height a and width 2a. The refractive index of the guide and of the substrate is respectively 1.5 and 1.45. Using the vectorial finite element program described in previous paragraph and a division consisting of 9 fourth order triangular elements, we find following points of the dispersion characteristic of the lowest mode :

HE mode : $k_o a = 11.4512$
$\beta a = 16.9478$

EH mode : $k_o a = 11.7012$
$\beta a = 17.3178$

The scalar approximation for the HE modes is based on the following functional :

$$L = \iint \left[\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2 - k_o^2 n^2 \phi^2 + \beta^2 \phi^2\right] dS \qquad (2)$$

This functional has the continuity of $\frac{\partial\phi}{\partial n}$ as natural boundary conditions. A finite element program based on functional (2) yields β as the eigenvalue of the matrix equation for a given k_o. In the case of an infinite slab guide equation (2) gives an exact variational expression for the TE slab modes. If we consider again the rectangular overlay guide one finds following points of the dispersion characteristic :

$k_o a = 11.4512$
$\beta a = 16.9481$

This is almost identical to the result obtained for the HE mode with the full vectorial analysis.
The scalar approximation for the EH modes is based on the following functional :

$$L = \iint \left[\frac{1}{n^2} \left(\frac{\partial\phi}{\partial x}\right)^2 + \frac{1}{n^2} \left(\frac{\partial\phi}{\partial y}\right)^2 + \frac{\beta^2}{n^2}\phi^2 - k_o^2\phi^2 \right] dS \qquad (3)$$

This functional has the continuity of $\left(\frac{1}{n^2}\frac{\partial\phi}{\partial n}\right)$ as natural boundary condition. A finite element program based on this functional yields k_o as eigenvalue of the matrix equation for a given β. In the case of an infinite slab guide, equation (3) gives an exact variational expression for the TM slab modes. Considering again the previous rectangular overlay guide, one finds following points on the dispersion characteristic :

$k_o a = 11.7086$

$\beta\ a = 17.3178$

This is almost identical to the result obtained for the EH modes with the full vectorial analysis.

From those examples one can see that the two scalar finite element formulations form an excellent approximation for the HE and EH type modes of the optical waveguide, even in the case where the width to height ratio of the guide is small. The main advantages of this scalar approximation are :

1. The dimensions of the matrices are reduced by a factor of 2, which means a reduction of the computer time by approximately a factor of 4.
2. The two scalar functionals are positive definite (or can immediately be made positive definite). All the eigenvalues are therefore positive and each one corresponds to a physical mode of the guide. This means that one can now easily compute the higher order modes of the guide.

To illustrate the use of those scalar finite element approximations, a number of modes of two different waveguides have been calculated. First we consider again the rectangular overlay guide described earlier. In fig. 3 contour plots are shown for the 5 lowest order HE modes of the guide. The normalised wavenumber $k_o a$ for all modes is equal to 25. As a second example we consider the trapezoidal overlay guide shown in fig. 4. Such a guide is obtained when the guide material is not completely etched away so that a thin layer remains over the complete surface of the waveguide. One can see how the four lowest order modes of such a guide are easily computed using only 12 fourth order elements.

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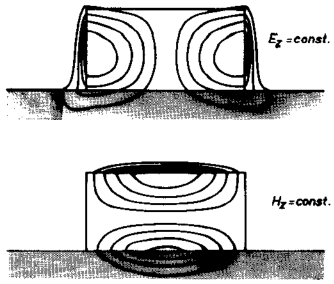

Fig. 2 Contour lines for E_z and H_z for the lowest order mode of a rectangular overlay guide

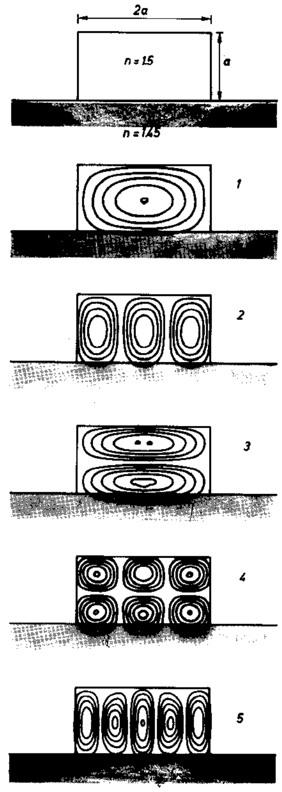

Fig. 3 Five lowest order modes of rectangular overlay guide

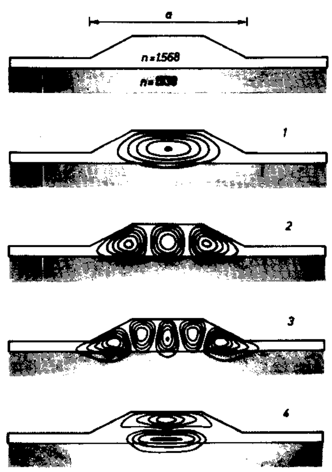

Fig. 4 Four lowest order modes of trapezoidal guide